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Are SC-FDE Systems Robust to CFO?

Ying Chen * , Jian (Andrew) Zhang †, A.D.S. Jayalath‡
*Institute for Telecommunication Research,
University of South Australia, Adelaide, SA5095, Australia
Email: Ying.Chen@unisa.edu.au
† Networked System, NICTA Australia
Email: Andrew.Zhang@nicta.com.au
‡ School of Engineering Systems
The Queensland University of Technology, Brisbane, QLD4001 Australia
Email: Dhammika.Jayalath@qut.edu.au

***Abstract*—This paper investigates the impact of carrier frequency offset (CFO) on Single Carrier wireless communication systems with Frequency Domain Equalization (SC-FDE). We show that CFO in SC-FDE systems causes irrecoverable channel estimation error, which leads to inter-symbol-interference (ISI). The impact of CFO on SC-FDE and OFDM is compared in the presence of CFO and channel estimation errors. Closed form expressions of signal to interference and noise ratio (SINR) are derived for both systems, and verified by simulation results. We find that when channel estimation errors are considered, SC-FDE is similarly or even more sensitive to CFO, compared to OFDM. In particular, in SC-FDE systems, CFO mainly deteriorates the system performance via degrading the channel estimation. Both analytical and simulation results highlight the importance of accurate CFO estimation in SC-FDE systems.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) and Single Carrier with Frequency Domain Equalization (SC-FDE) transmission schemes are two competitive solutions for future wireless broadband communication systems. They are already being proposed for broadband systems such as WiMax and 3GPP LTE. Similar to OFDM, SC-FDE enables simple frequency domain equalization in dense multipath channels. One major advantage of SC-FDE is that SC-FDE signal has much lower peak-to-average-power ratio than OFDM signal, and it is generally believed that, robustness to carrier frequency offset (CFO) is SC-FDE's another advantage [1] [2].

CFO is caused by the frequency mismatch between the local oscillators at the transmitter and receiver and the Doppler frequency shift due to relative motion between transmitter and receiver. In OFDM systems, CFO causes distortions of the orthogonality among subcarriers and introduces inter-carrier-interference (ICI) which could lead to severe performance degradation [3]–[5]. The CFO impact on SC-FDE systems has been studies in [1], [2], [6]–[8], where the analysis is based on either AWGN channel or perfect channel estimation. The simplified models lead to the result that CFO only causes phase shift in SC-FDE systems and thus SC-FDE is robust to CFO. In this paper, however, we will show that in a practical SC-FDE system, CFO inevitably causes channel estimation errors even in the absence of AWGN, and inter-symbol interference (ISI) will then be introduced, degrading the system performance severely. Furthermore, channel estimation errors increase with increasing CFO and hence performance decreases. The impact of CFO on SC-FDE and OFDM will be compared within a common framework, based on the measurement of signal-to-noise-interference ratio (SINR). SINR cannot accurately describe the system performance, however, it is a simple way to evaluate how severe the interference is.

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The rest of the paper is organized as follows. In Section II, mathematical models are formulated for SC-FDE and OFDM systems incorporating CFO terms. In Section III, channel estimation errors and ISI due to CFO are investigated. In Section IV, closed-form expression of SINR is derived, and the impact of CFO on SC-FDE and OFDM is analyzed and compared. Simulation results are shown in Section V. Finally, Section VI concludes the paper.

Notations: $\vec{x}$ for a vector x, capital symbol for frequency domain signal, bold capital symbol for matrix, the superscript $^{-1}$ for matrix inverse and $\text{diag}(\vec{x})$ for a diagonal matrix with $\vec{x}$ being the diagonal elements.

II. System Model

In SC-FDE systems, symbols to be transmitted are organized in blocks, and a guard interval is appended to each block. The guard interval could be padded by zero or cyclic prefix. For simplicity, zero-padded (ZP) FDE systems is considered in this paper. In the receiver, a frequency domain equalizer is applied to remove the channel effect, and an inverse fast fourier transform (IFFT) is applied to recover the symbols. For detailed information of SC-FDE systems, the readers are referred to [6]. An SC-FDE system can be regarded as obtained from an OFDM system by shifting the IFFT module from the transmitter to the receiver. Thus they can be easily formulated in a common mathematical model, as done in this paper.

Denote a block of M symbols to be transmitted in time domain as $\vec{x} = [x_0, x_1, \dots, x_{M-1}]^T$, and their frequency domain dual as $\vec{X} = [X_0, X_1, \dots, X_{M-1}]^T$. Let the channel impulse response be $\vec{h} = [h_0, h_1, \dots, h_{L-1}]^T$. Without loss of generality, we assume that $M \gg L$ and L equals to the length of the guarding interval (channel could be extended with zeros if it is shorter).

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Consider a CFO term ω_d in the receiver, normalized with respect to the signal bandwidth. Since accumulated phase shift only causes a fixed phase shift for all samples within a block, we will ignore its effect in the following analysis. In a ZP-FDE system, overlapping and sum (O&S) [9] is implemented in the receiver to convert a linear convolution to a circular convolution between the transmitted signal and the channel. For every block of length $M+L$, the O&S operation adds the last L samples of the received signal to the first L samples. Denote the $M+L$ received samples as $\vec{y}_0$, which is given by

$$\vec{y}_0 = \begin{pmatrix} p_0 & 0 & \dots & 0 \\ 0 & p_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_{M+L-1} \end{pmatrix} \begin{pmatrix} h_0 & 0 & \dots & 0 \\ h_1 & h_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_0 \end{pmatrix} \begin{pmatrix} \vec{x} \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \vec{w}_0, \quad (1)$$

where the elements $p_k = e^{-jk\omega_d}, \quad p_0 = 1$ in the diagonal matrix are phase shifting terms due to the CFO, and $\vec{w}_0$ denotes the AWGN samples. The M samples obtained after O&S operation can be written as

$$\begin{aligned} \vec{y} &= \mathbf{HP}\vec{x} + \vec{w} \quad (2) \\ &= \begin{pmatrix} h_0 & 0 & \dots & p_1 h_1 \\ p_1 h_1 & h_0 & \dots & p_2 h_2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_1 \end{pmatrix} \begin{pmatrix} p_0 & 0 & \dots & 0 \\ 0 & p_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_{M-1} \end{pmatrix} \vec{x} + \vec{w}, \end{aligned}$$

where $\mathbf{H}$ is a circulant channel matrix, absorbing part of the CFO terms, $\mathbf{P} = \text{diag}[p_0, p_1, \dots, p_{M-1}]$ is a diagonal matrix containing another part of CFO terms, and $\vec{w}$ is the noise vector.

III. Impact of CFO on FDE system

In this section, we will investigate how an FDE system is affected by CFO when an estimated channel matrix rather than a perfect one is used in the equalization.

A. Interference due to CFO

It is well known that a circulant matrix can be diagonalized by a pair of FFT and IFFT matrices. FDE exploits this property to realize a one-tap equalizer in the frequency domain. Applying an FFT to $\vec{y}$ in (2), we get

$$\vec{Y} = \mathbf{F}\vec{y} = \mathbf{DC}\vec{X} + \mathbf{F}\vec{w} \quad (3)$$

where $\mathbf{F}$ denotes the FFT matrix, $\vec{X} = \mathbf{F}\vec{x}$, $\mathbf{D} = \mathbf{FHF}^H = \text{diag}[d_0, \dots, d_{M-1}]$ is the diagonal frequency-domain channel matrix, with ^{H} denoting hermitian conjugate, and $\mathbf{C} = \mathbf{FPF}^H$ represents the CFO terms in frequency domain. $\mathbf{C}$ is a circulant matrix with its first column $[c_0, \dots, c_{M-1}]^T$ equaling to the Fourier Transform of $[p_0, \dots, p_{M-1}]^T$, and $\sum_{j=0}^{M-1} |c_j|^2 = 1$.

Denote the estimated channel matrix as $\hat{\mathbf{D}}$ and the corresponding time domain estimate of $\mathbf{H}$ as $\hat{\mathbf{H}}$, where $\hat{\mathbf{D}}$ is a diagonal matrix, and $\hat{\mathbf{H}}$ is a circulant matrix. When a Zero Forcing (ZF) equalizer is applied, the estimate of $\vec{X}$ becomes

$$\hat{X} = \hat{\mathbf{D}}^{-1}\mathbf{DC}\vec{X} + \hat{\mathbf{D}}^{-1}\mathbf{F}\vec{w} = \mathbf{G}\vec{X} + \hat{\mathbf{D}}^{-1}\mathbf{F}\vec{w}, \quad (4)$$

where $\mathbf{G} = \hat{\mathbf{D}}^{-1}\mathbf{DC}$. Equation (4) actually gives the estimated symbols in a ZP-OFDM system. Since $\mathbf{C}$ is not a diagonal matrix, there is ICI caused by CFO in ZP-OFDM systems.

For an SC-FDE system, the estimated signal in the time domain is given by

$$\hat{x} = \mathbf{F}^H\mathbf{GF}\vec{x} + \mathbf{F}^H\hat{\mathbf{D}}^{-1}\mathbf{F}\vec{w} = \mathbf{\Gamma}\vec{x} + \mathbf{F}^H\hat{\mathbf{D}}^{-1}\mathbf{F}\vec{w}, \quad (5)$$

where $\mathbf{\Gamma} = \mathbf{F}^H\mathbf{GF}$. When there are channel estimation errors, $\hat{\mathbf{D}}^{-1}\mathbf{D}$ will not be an identity matrix, and $\mathbf{\Gamma}$ will not be a diagonal matrix. So there will be ISI in SC-FDE systems. Although ISI is directly caused by channel estimation errors, we show next that CFO causes channel estimation errors even in the absence of noise, and channel estimation errors increase with increasing CFO.

B. Channel Estimation Errors due to CFO

In this subsection, we investigate the variance of channel estimation error at k^{th} subcarrier, $\sigma_e(k)$, in the presence of residual CFO ω_d and AWGN with variance σ_w^2. Denote a block of training sequence used for channel estimation as $\vec{X}_p = [X_p(0), X_p(1), \dots, X_p(M-1)]$. According to (3), the corresponding received frequency domain signal $\vec{Y}_p$ is given by

$$\vec{Y}_p = \mathbf{DC}\vec{X}_p + \mathbf{F}\vec{w}. \quad (6)$$

So for the k^{th} subcarrier, we have

$$Y_p(k) = c_0 d_k X_p(k) + d_k \sum_{i=0, i\neq k}^{M-1} c_{(i-k)_M} X_p(i) + w(k) \quad (7)$$

where $(i-k)_M$ denotes $i-k$ modulo M. Assuming $|X_p(i)| = 1$, for least square channel estimation, the estimated channel response at the k^{th} subcarrier can be represented as

$$\begin{aligned} \hat{d}_k &= X_p^*(k) Y_p(k) \\ &= c_0 d_k + d_k \sum_{i=10, i\neq k}^{M-1} c_{(i-k)_M} X_p(i) X_p^*(k) + w(k) X_p^*(k) \\ &= c_0 d_k + e_d(k), \quad (8) \end{aligned}$$

where

$$c_0 = \frac{e^{-j\frac{M+1}{2}\omega_d} \sin(\frac{M\omega_d}{2})}{\sin(\frac{\omega_d}{2})} \quad (9)$$

equals to the diagonal elements in $\mathbf{C}$, and

$$e_d(k) = d_k \sum_{i=0, i\neq k}^{M-1} c_{(i-k)_M} X_p(i) X_p^*(k) + w(k) X_p^*(k) \quad (10)$$

is the estimation error caused by AWGN noise & ICI. The coefficient c_0 will be absorbed into the channel estimates. Then the variance of channel estimation error at the k^{th} subcarrier equals to the variance of $e_d(k)$. Assume that $\vec{X}_p$

and $\vec{w}$ are statistically independent, and both have zero mean, the variance of $e_d(k)$ can be calculated as

$$\sigma_e^2(k) = E\{e_d^*(k)e_d(k)\} = \left(\left|\sum_{i=0,i\neq k}^{M-1} c_{(i-k)_M} X_p(i)\right|^2\right) |d_k|^2 + \sigma_w^2. \quad (11)$$

When Q identical training symbols are used for channel estimation, $\sigma_e^2(k)$ becomes

$$\sigma_e^2(k) = \frac{\sigma_w^2}{Q} + \left|d_k \sum_{i=0,i\neq k}^{M-1} c_{(i-k)_M} X_p(i)\right|^2. \quad (12)$$

where we have used the fact that identical transmitted signal causes identical ICI and $w(k)$ is AWGN.

Equation (12) shows that the variance of the channel estimation error due to the CFO is independent of the number of identical training symbols. In other words, channel estimation error due to CFO will cause some error floor and cannot be reduced by increasing the number of identical training symbols. Now, if we treat $X_p(k)$ as independently and identically distributed (i.i.d.) variables with zero mean and variance σ_X^2, using $\sum_{i=0,i\neq k}^{M-1} |c_{(i-k)_M}|^2 = 1 - |c_0|^2$, we can further get

$$\sigma_e^2(k) = \sigma_w^2/Q + (1-|c_0|^2)|d_k|^2\sigma_x^2 = \frac{\sigma_w^2}{Q} + \sigma_x^2\big(1 - \left|\frac{\sin(M\omega_d/2)}{\sin(\omega_d/2)}\right|^2\big)|d_k|^2. \quad (13)$$

Equation (13) indicates that CFO introduces an interference term with power $(|(1 - \left|\frac{\sin(M\omega_d/2)}{\sin(\omega_d/2)}\right|^2)|d_k|^2$ into the channel estimation error. It is easy to verify that when $\omega_d < 2\pi/M$, the subcarrier interval, the interference power increases with increasing ω_d.

IV. PERFORMANCE DEGRADATION DUE TO CFO

In this section we will develop closed-form expression of SINR to evaluate the impact of CFO on SC-FDE and OFDM systems. The SINR is defined as the ratio between the expectation of signal power and the expectation of noise and interference power. The expectation operation $E\{\cdot\}$ in the derivation below is with respect to the transmitted symbols.

A. SINR Degradation in OFDM systems

In the OFDM system, from (4) the efficient signal power at i^{th} subcarrier is given by

$$P_S(i) = E\{|(\hat{d}_i^{-1}c_0 d_i X(i))|^2\} = \frac{c_0^2|d_i|^2}{|\hat{d}_i|^2}\sigma_X^2, \quad (14)$$

where we have used the assumption that the transmitted symbols are i.i.d variables with zero mean and variance σ_X^2, and $\hat{d}_i$ is a known constant for i^{th} subcarrier. The interference power can be computed as

$$P_I(i) = E\left\{\left|\sum_{j=0,j\neq i}^{M-1} \frac{c_{(j-i)_M} d_i X(j)}{\hat{d}_i}\right|^2\right\} = \frac{\sum_{j=0,j\neq i}^{M-1} |c_{(j-i)_M}|^2 |d_i|^2}{|\hat{d}_i|^2}\sigma_X^2, \quad (15)$$

The SINR at the i^{th} subcarrier is thus given by

$$\mathrm{SINR_{OFDM}}(i) = \frac{P_S(i)}{\left(P_I(i) + \frac{\sigma_w^2}{|\hat{d}_i|^2}\right)} = \frac{|c_0|^2|d_i|^2}{|d_i|^2 \sum_{j=0,j\neq i}^{M-1} |c_{(j-i)_M}|^2 + \frac{\sigma_w^2}{\sigma_X^2}} = \frac{|c_0|^2}{1-|c_0|^2 + \frac{\sigma_w^2}{|d_i|^2\sigma_X^2}}. \quad (16)$$

From (16) we can see that the SINR in the OFDM system is robust to channel estimation error.

B. SINR Degradation in SC-FDE systems

In the SC-FDE system, the symbol estimates can also be represented as $\hat{x} = \hat{\mathbf{H}}^{-1}\mathbf{HP}\vec{x} + \hat{\mathbf{H}}^{-1}\vec{w}$, where $\hat{\mathbf{H}}^{-1}\mathbf{H}$ is a circulant matrix and $\mathbf{P}$ is a diagonal matrix with the magnitude of diagonal elements being 1. Thus it is easy to verify that statistically, each estimated symbol sees same efficient signal power and interference power. So to characterize the SINR for SC-FDE systems, we can calculate the SINR for any data symbol.

According to (5), for every estimate, the efficient signal power can be represented as

$$P_S = \frac{1}{M}E\{\sum_{i=0}^{M-1} |\gamma_{i,i}x_i|^2\} = \frac{\sigma_X^2}{M}E\{\sum_{i=0}^{M-1}|\gamma_{i,i}|^2\}, \quad (17)$$

where $\gamma_{i,i}$ is the i^{th} diagonal element of $\mathbf{\Gamma}$.

Since $\mathbf{\Gamma} = \mathbf{F}^H\mathbf{GF} = (\mathbf{F}(\mathbf{FG})^H)^H$, $\gamma_{i,i}$ can be calculated as

$$\gamma_{i,i} = \frac{1}{M}\sum_{k=0}^{M-1}\left(\sum_{v=1-k}^{M-k} g_{v+k,k} e^{-j\frac{2\pi}{M}iv}\right) = \sum_{v=0}^{M-1}\left(\frac{1}{M}\sum_{k=0}^{M-1} g_{(v+k)_M,k}\right)e^{-j\frac{2\pi}{M}iv}, \quad (18)$$

where $g_{(v+k)_M,k} = \hat{d}_{(v+k)_M}^{-1} d_{(v+k)_M} c_v$ is the element of $\mathbf{G}$ at the $(v+k)_M$-th row and k-th column.

As $|\gamma_{i,i}|$ can be viewed as the FFT coefficients of $\frac{1}{\sqrt{M}}\sum_{k=0}^{M-1} g_{(v+k)_M,k}$, and FFT does not change the signal power, we can get

$$\sum_{i=0}^{M-1}|\gamma_{i,i}|^2 = \frac{1}{M}\sum_{v=0}^{M-1}\left|\sum_{k=0}^{M-1}\frac{d_{(k+v)_M}}{\hat{d}_{(k+v)_M}}c_v\right|^2 = \frac{1}{M}\left(\sum_{v=0}^{M-1}|c_v|^2\right)\left|\sum_{k=0}^{M-1}\frac{d_k}{\hat{d}_k}\right|^2 = \frac{1}{M}\left|\sum_{k=0}^{M-1}\frac{d_k}{\hat{d}_k}\right|^2. \quad (19)$$

According to (13), the variance of channel estimation error only depends on the magnitude of the specific channel. Thus we can assume channel estimation errors are independent random variables with zero mean and variance $\sigma_e^2(k)$ given by (13), and we get

$$E\left\{\frac{d_j}{\hat{d}_j}\frac{d_k^*}{\hat{d}_k^*}\right\} = \frac{1}{|c_0|^2} \tag{20}$$

for $k \neq j$. The efficient signal power P_S can be further written as

$$P_S = \left(\frac{1}{M^2}\sum_{k=0}^{M-1} E\left\{\left|\frac{d_k}{\hat{d}_k}\right|^2\right\} + \frac{M-1}{M|c_0|^2}\right)\sigma_X^2, \tag{21}$$

where

$$E\left\{\left|\frac{d_k}{\hat{d}_k}\right|^2\right\} = E\left\{\frac{|d_k|^2}{|c_0 d_k + e_d(k)|^2}\right\}$$
$$= \frac{1}{|c_0|^2}E\left\{\frac{1}{\left(1+\frac{e_d(k)c_0^* d_k^* + e_d^*(k)c_0 d_k + |e_d(k)|^2}{|c_0 d_k|^2}\right)}\right\}.$$

Generally, $e_d(k)$ is relatively small compared to $c_0 d_k$. Thus $\beta = -(e_d(k)c_0^* d_k^* + e_d^*(k)c_0 d_k + |e_d(k)|^2)/|c_0 d_k|^2$ is much smaller than one. Representing the function $1/(1-\beta)$ as Taylor series and truncating it to the second order, $E\{|d_k/\hat{d}_k|^2\}$ can be approximated as

$$E\left\{\left|\frac{d_k}{\hat{d}_k}\right|^2\right\} \approx \frac{1}{|c_0|^2}\left(E\left\{1+\left|\frac{e_d(k)}{d_k c_0}\right|^2 + 2\mathcal{R}\left\{\left(\frac{e_d(k)}{d_k c_0}\right)^2\right\}\right\}\right)$$
$$= \frac{1}{|c_0|^2}\left(1+\frac{\sigma_e^2(k)}{|d_k c_0|^2}\right), \tag{22}$$

where $\mathcal{R}\{\cdot\}$ denotes the real part of a complex variable. In the above derivation, we have used the assumption that the real and imaginary parts of $e_d(k)$ are independent and identically distributed, so that $E\{\mathcal{R}(e_d^2(k))\} = 0$.

The efficient signal power can then be approximated as

$$P_S \approx \left(\frac{1}{|c_0|^2} + \frac{1}{M^2}\sum_{k=0}^{M-1}\frac{\sigma_e^2(k)}{|d_k|^2|c_0|^4}\right)\sigma_X^2. \tag{23}$$

The interference power can be calculated by removing the power of the efficient signal and noise from the power of the observation symbol. According to $\mathbf{\Gamma} = \mathbf{F}^H\mathbf{G}\mathbf{F}$, the power of the observation symbol can be computed as

$$\frac{1}{M}E\{\mathrm{Tr}(\mathbf{x}^H\mathbf{\Gamma}^H\mathbf{\Gamma}\mathbf{x})\} = \frac{1}{M}E\{\mathrm{Tr}(\mathbf{\Gamma}^H\mathbf{\Gamma})\}\sigma_X^2 \tag{24}$$
$$= \frac{1}{M}\left(\sum_{j=0}^{M-1}|c_j|^2\right)E\left\{\sum_{k=0}^{M-1}|\frac{d_k}{\hat{d}_k}|^2\right\}\sigma_X^2$$
$$= \frac{1}{M}E\left\{\sum_{k=0}^{M-1}|\frac{d_k}{\hat{d}_k}|^2\right\}\sigma_X^2,$$

where $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix.

According to (22), (23) and (24), the interference power is given by

$$P_I \approx \frac{M-1}{M^2}\sum_{k=0}^{M-1}\frac{\sigma_e^2(k)}{|d_k|^2|c_0|^4}\sigma_X^2, \tag{25}$$

and the SINR becomes

$$\mathrm{SINR}_{\mathrm{SC}} = \frac{M^2|c_0|^2 + \sum_{k=0}^{M-1}\frac{\sigma_e^2(k)}{|d_k|^2}}{(M-1)\sum_{k=0}^{M-1}\frac{\sigma_e^2(k)}{|d_k|^2} + M\frac{\tilde{\sigma}_w^2|c_0|^2}{\sigma_X^2}}, \tag{26}$$

where $\tilde{\sigma}_w^2 = \sigma_w^2(\sum_{k=0}^{M-1}E\{|\hat{d}_k|^{-2}\})$ is the noise power after frequency domain equalization.

From (16), we know that in OFDM systems, the SINR is linked to the CFO directly and does not depend on the estimated channel $\hat{\mathbf{D}}$. For SC-FDE systems, (26) shows that the SINR is a function of $\sigma_e^2(k)$ and the CFO , and it degrades with $\sigma_e^2(k)$ increasing. When there is no channel estimation errors and $\sigma_e^2(k) = 0$, we get $P_I = 0$ which means there is no ISI caused by CFO. However, from (12), we have seen that, in SC-FDE, CFO inevitably introduces channel estimation errors and larger CFO leads to larger variance of channel estimation errors and lower SINR. So CFO in SC-FDE systems impacts the performance both directly and indirectly via channel estimation errors, and it is hence important to achieve lower CFO in SC-FDE systems.

V. Simulation Results

Zero-padded OFDM and SC-FDE systems are simulated to verify the above analytical results. In both systems, the number of subcarriers is $M = 64$, the length of guarding interval is $L = 16$, and ETSI Multipath A [10], an indoor channel model, is adopted. The CFO values presented in the simulation results are normalized with respect to the subcarrier interval. Accumulated CFO per block, is assumed to be known and compensated in the simulation. Unless stated otherwise, channel is estimated with two identical training symbols with a frequency domain least square approach as specified in (8).

Fig. 1 shows the SIR degradation due to CFO in uncoded SC-FDE and OFDM systems with BPSK modulation. As shown in [1], OFDM and SC systems have different anti-noise capability, and SC-FDE suffers from the so-called noise enhancement and propagation problem. To focus on the SINR degradation caused by CFO, numerical and theoretical results are presented for noise-free case. For each ω_d, the channel estimation errors are deliberately set to be random variables with mean zero and variance specified in (13). In the figure, solid curves correspond to the Monte-Carlo simulation results, and dash-dot curves correspond to the theoretical results based on (16) and (26) with $\sigma_w^2 = 0$. The theoretical and numerical results match well in the figure. The figure also shows that, SC-FDE systems experience similar or slightly larger SINR degradation, compared to OFDM systems.

From Fig. 2 to 4, we show how the bit error rate (BER) changes with channel estimation under different conditions in both SC-FDE and OFDM systems with BPSK modulation. Three CFO values, 0, 0.064 and 0.192 are used for comparison. In simulations generating all the three figures, only channel estimation is implemented differently. Channel coefficients are obtained in the following ways: 1) perfectly known channel in Fig. 2; 2) estimated channel in the presence of only AWGN in Fig. 3; and 3) estimated channel in the

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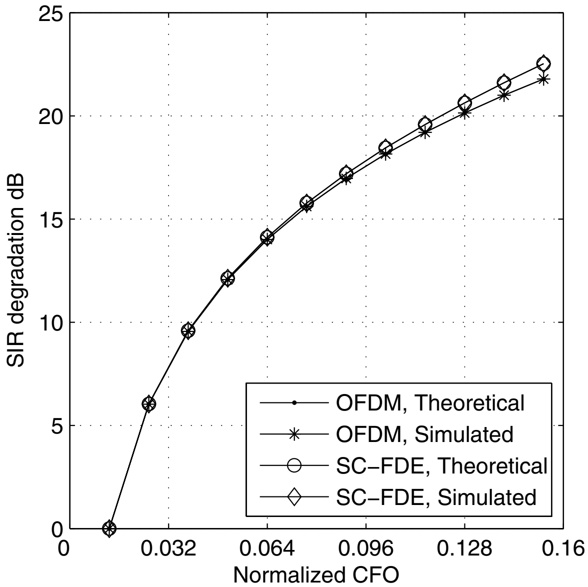

Fig. 1. SIR degradation in SC-FDE and OFDM systems versus normalized CFO. Curves are plotted as the difference with respect to the SIR degradation value corresponding to the first CFO = 0.0128

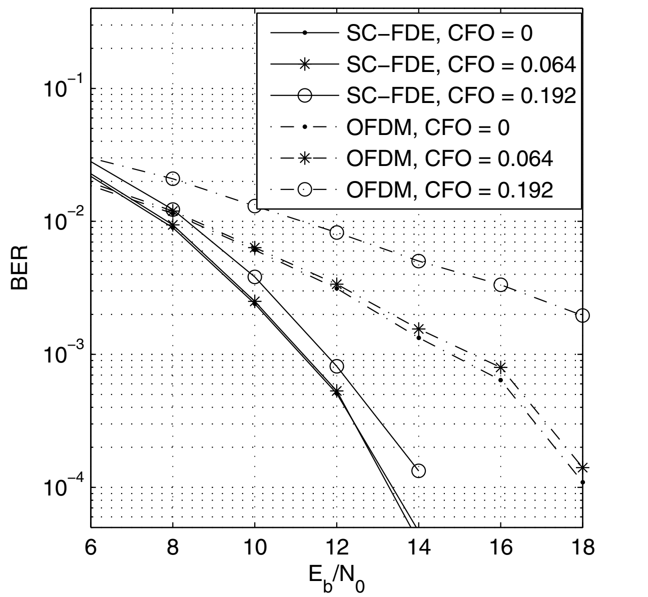

Fig. 2. BER of uncoded SC-FDE and OFDM systems with BPSK modulation and perfect channel coefficients

presence of both AWGN and CFO. From these three figures, we can get the following observations:

- As shown in Fig. 2, without any channel estimation error, CFO itself only has small impact on SC-FDE systems, while OFDM is sensitive to CFO. Furthermore, SC-FDE outperforms OFDM after a small Eb/N0 value;
- As shown in Fig. 3, with channel estimated in the presence of AWGN only, the performance of SC-FDE degrades significantly compared to the case with perfect channel knowledge. However, CFO only causes slight performance degradation, and performance degradation is mainly connected with channel estimation error;
- As shown in Fig. 4, when channel is estimated in the presence of CFO and AWGN, SC-FDE suffers significant performance loss and the performance loss increases notably with increasing CFO. Thus it becomes clear that CFO mainly deteriorates the system performance via degrading the channel estimation.
- From the three figures, we can see that OFDM only suffers slight performance degradation from channel estimation errors.

Fig. 5 shows how the BER performance is affected by CFO for SC-FDE and OFDM systems with 16QAM modulation. In the figure, channel is estimated in the presence of both CFO and AWGN. Compare Fig. 5 and 4, we can see that SC-FDE systems with higher order modulations are even more sensitive to CFO.

Fig. 6 shows how the BER performance changes with CFO with E_b/N_0 fixed at 17dB. Channel is estimated in the presence of CFO and AWGN. For both QPSK and 16QAM, we can see that compared to OFDM, the performance of SC-FDE degrades more rapidly with increasing CFO, particularly

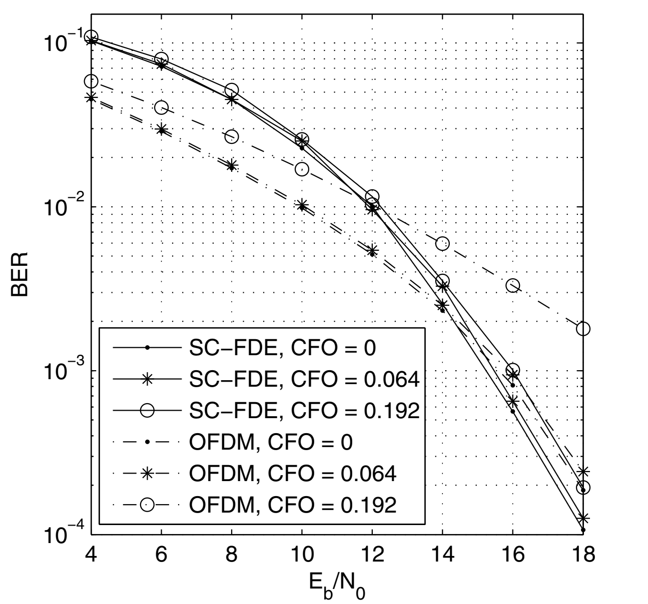

Fig. 3. BER of uncoded SC-FDE and OFDM systems with BPSK modulation and channel estimated in the presence of AWGN only

at smaller CFO. It is thus obvious that SC-FDE is even more sensitive to CFO than OFDM.

VI. CONCLUSIONS

In this paper, we have shown that CFO can significantly degrade the performance of SC-FDE systems. Both analytical and simulation results strongly agree that CFO can significantly increases the channel estimation errors, which directly

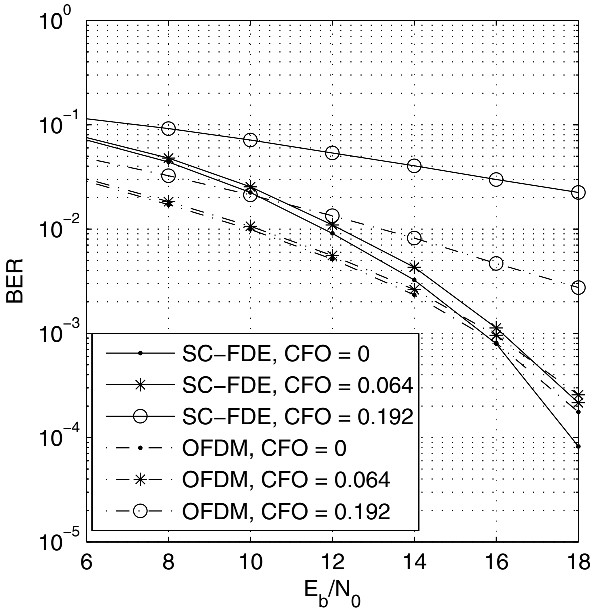

Fig. 4. BER of uncoded SC-FDE and OFDM systems with BPSK modulation and channel estimated in the presence of both CFO and AWGN

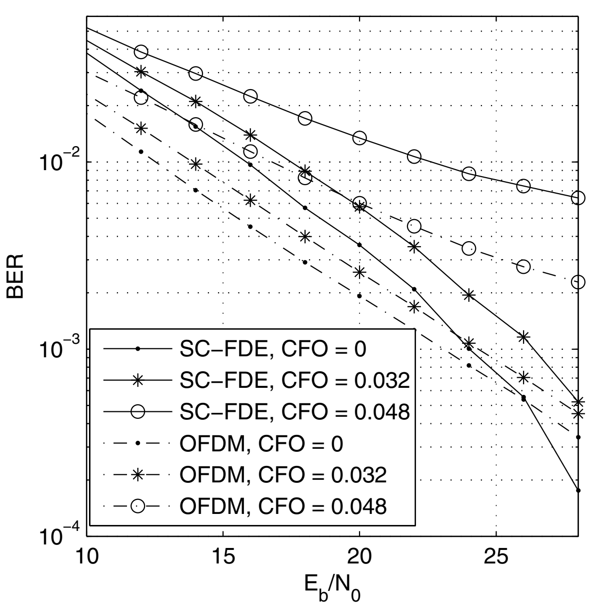

Fig. 5. BER of uncoded SC-FDE and OFDM systems with 16QAM modulation

introduce inter-symbol interference in SC-FDE systems and cause notable performance degradation. Performance loss is proportional to the CFO value. It has also been shown that SC-FDE is even more sensitive to CFO, compared to OFDM. Therefore it is evident that CFO effect should be minimized in channel estimation, and accurate carrier frequency synchronization can significantly improve the performance of practical SC-FDE systems.

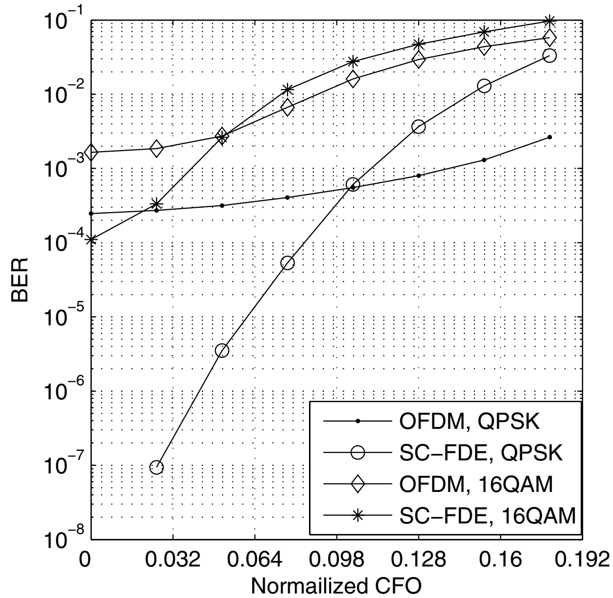

Fig. 6. BER of uncoded SC-FDE and OFDM systems with changing CFO at E_b/N_0=17dB

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